Chapter 3: Dependence Testing

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Important definitions

Subscript partitioning algorithms

Exact and inexact tests

ZIV, SIV, MIV tests

Dependence Testing Goals

Finding all dependences $\Leftarrow$ *required!*

Proving others independent $\Leftarrow$ *optional*

Taking advantage of information

- Vectorization
- Parallelization

Index variables

appear in looping constructs

Subscript expressions

actually refer to specific array locations

```
DO i = 1,N                  ⇐ i index
   A(i+2,i) = B(2*i)        ⇐ (i+2),(i),(2*i)
                              subscript expressions
```

Iteration Vector

- Ordered n-tuple
- Value of each index variable

Dependence Equations

$$\begin{aligned} \alpha &< \beta \\ f_i(\alpha) &= g_i(\beta), \forall i \end{aligned}$$

f and g subscript functions,

α and β iteration vectors,

i varies over subscripts

Induction variables

linear functions of loop indices

```
j = 2
DO  i = 1,N
    j = j + 2
    A(j) = A(j-1)
ENDDO
```

```
DO i = 1,N
   A(2i+2) = A(2i+1)
ENDDO
j = 2N + 2
```

Assume induction vars replaced

Undecidable for arbitrary subscripts

Conservative testing only!

Possible for simple subscripts

- Exact tests NP-complete
- Efficient conservative tests ok

Zero Index Variable — ZIV

Iteration-independent value

Single Index Variable — SIV

Linear in one index

`2*i + 4`

Multiple Index Variable — MIV

Linear in multiple indices

`2*i + 4*j + 2`

Test for dependence on each subscript

Conservative test

```
DO i
   A(i+1,i+2) = A(i,i) + C
ENDDO
```

Separable subscript expression

- Indices used in only this position
- Can test this position independently

```
DO i
   DO j
      DO k
         A(i, j, j) = A(i, j, k) + C
```

i separable, j and k coupled

```
DO i
   DO j
      DO k
         A(i+1, j+2, k-1) = A(i, j+1, k) + C
```

All indices separable

Partitioning subscripts

- Separable
- Minimal coupled groups

Partition subscripts

Classify ZIV/SIV/MIV

Test each partition

Merge direction vectors

Cartesian Product

$merge$	$(<,=)$	$(<,<)$
$(<)$	$(<,<,=)$	$(<,<,<)$
$(=)$	$(=,<,=)$	$(=,<,<)$

Subtract subscript expressions

Nonzero $\Rightarrow$ independent

Based on linear Diophantine

$a_1 i + c_1$ and $a_2 i + c_2$

Strong SIV

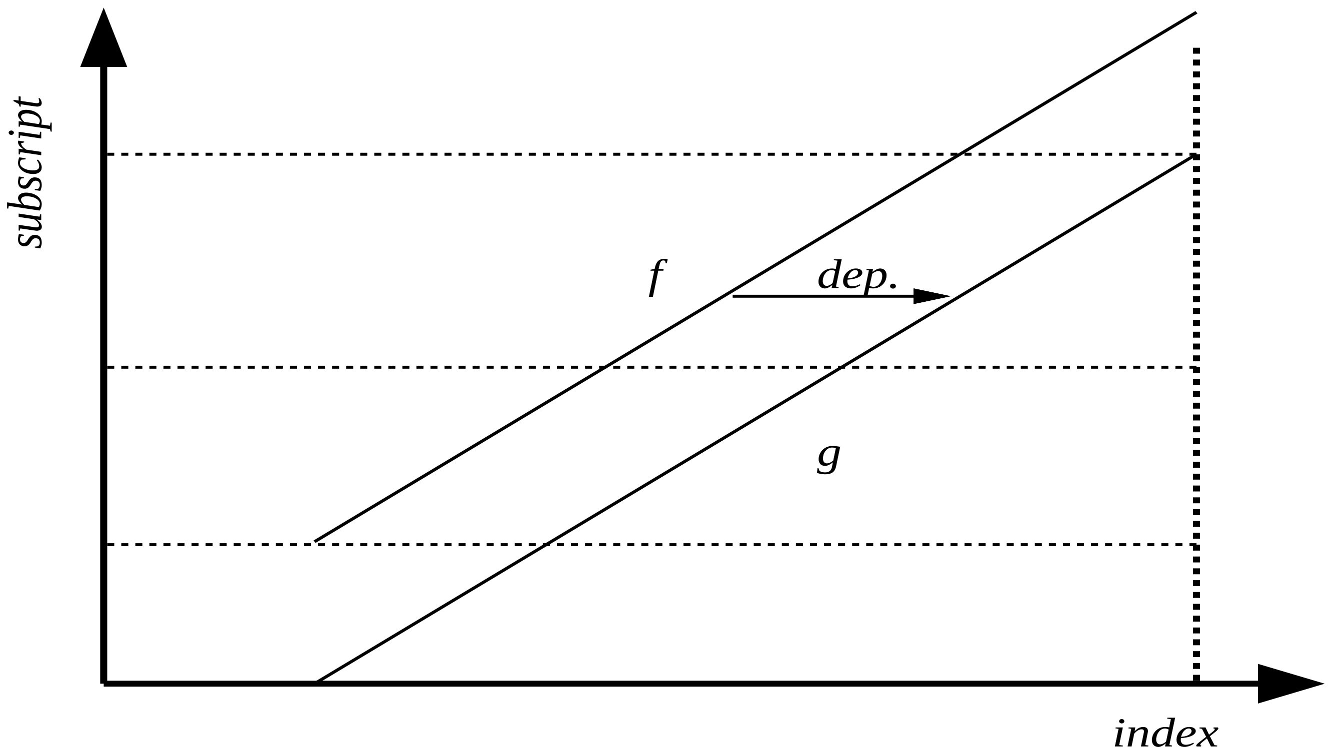
$a_1 = a_2$

Weak SIV

$a_1 \neq a_2$

- Weak Zero : $a_2 = 0$ or $a_1 = 0$
- Weak Crossing : $a_1 = -a_2$

Strong SIV



$d = i' - i = \frac{c_1 - c_2}{a}$

Dependence only if d integer and $|d| \leq U - L$

Strong SIV

```
DO i = 1,N
   A(i+2N) = A(i+N)+C
ENDDO
```

$$
\begin{aligned}
d &= \frac{2N - N}{1} \\
&= N \\
N &> N - 1
\end{aligned}
$$

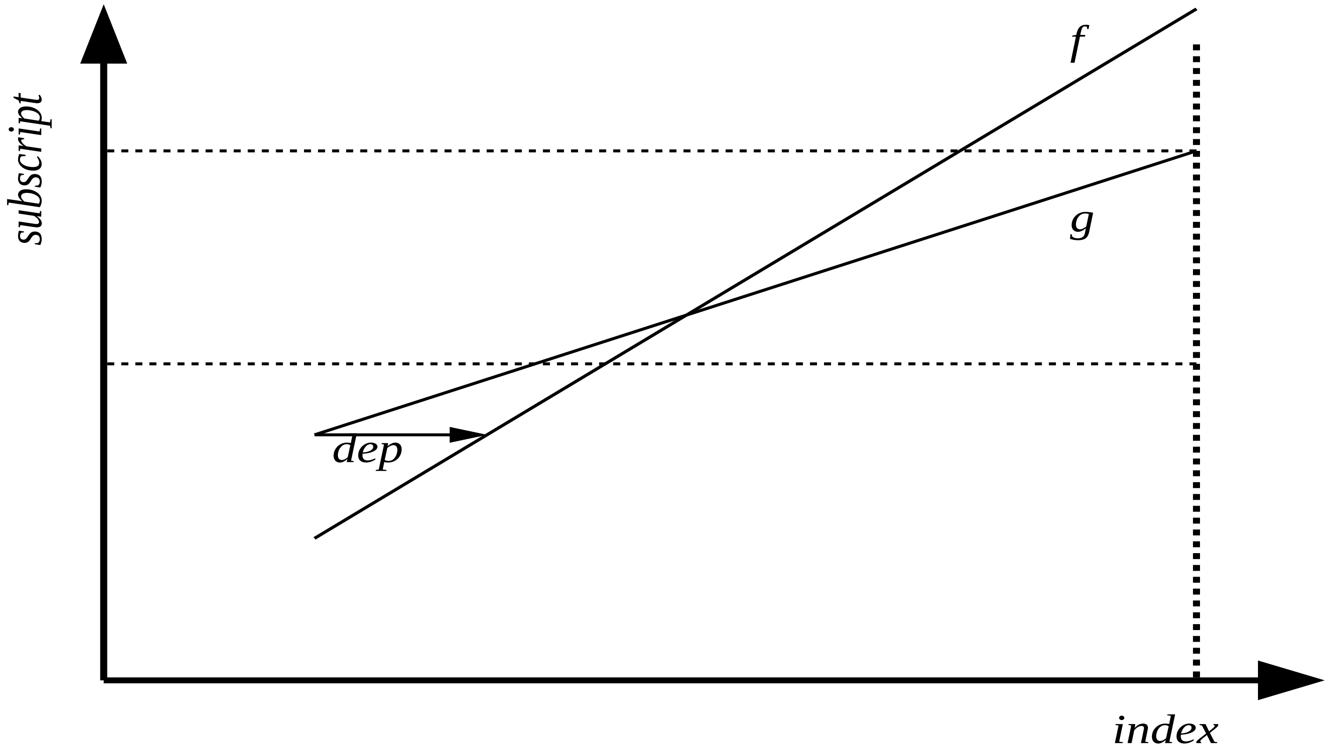
No dependence!

```
DO i = 1,99
   A(i+1) = A(i)+C
ENDDO
```

$$
\begin{aligned}
d &= \frac{1}{1} \\
&= 1 \\
1 &\leq 98
\end{aligned}
$$

Dependence!

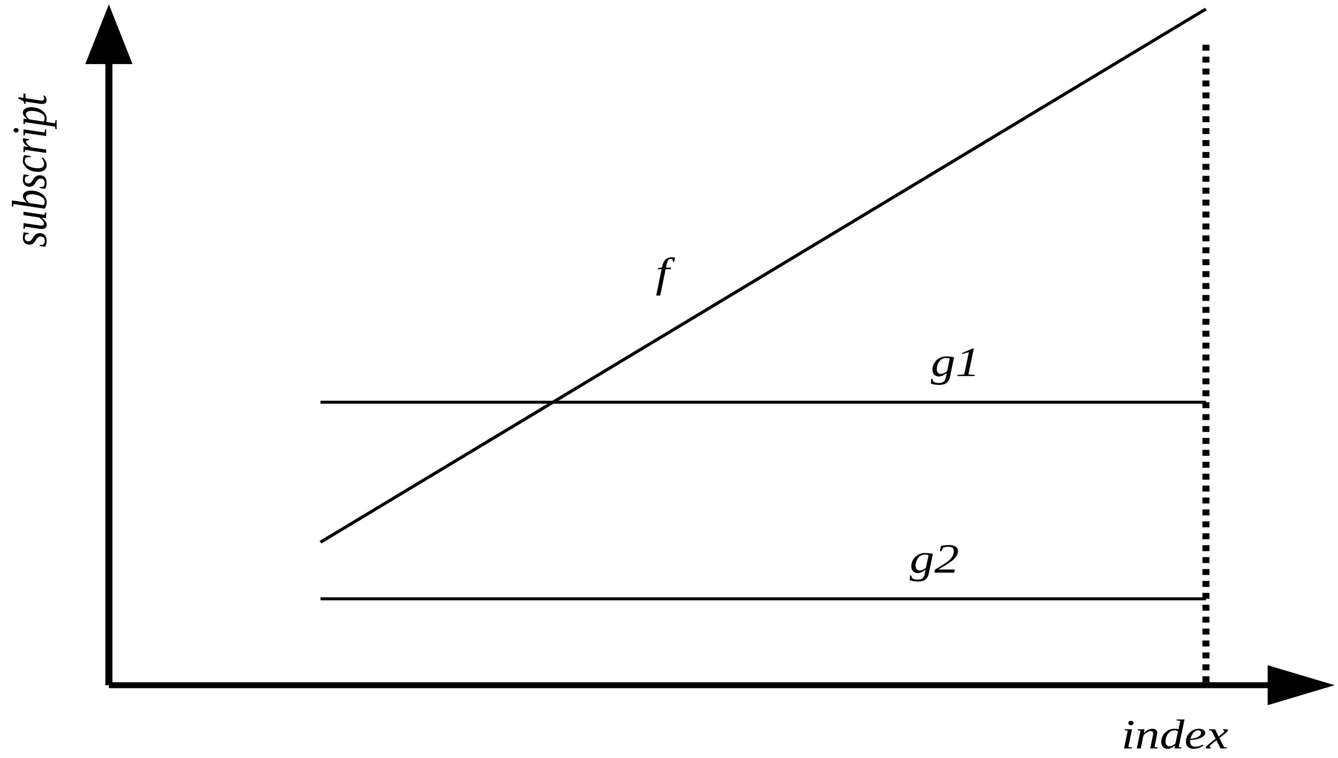
Weak SIV



$$a_1 i + c_1 = a_2 i^{'} + c_2$$

Dependence only if $\exists$ int. sol'n $(L \leq i, i^{'} \leq U)$

Weak Zero SIV



$i = \frac{c_2 - c_1}{a_1}$

Dependence only if i int. and $L \leq i \leq U$

Weak Zero SIV

```
DO i = 1,N
   Y(i,N) = Y(1,N)+Y(N,N)
ENDDO
```

$$i = 1, N$$

$$1 \leq 1 \leq N$$

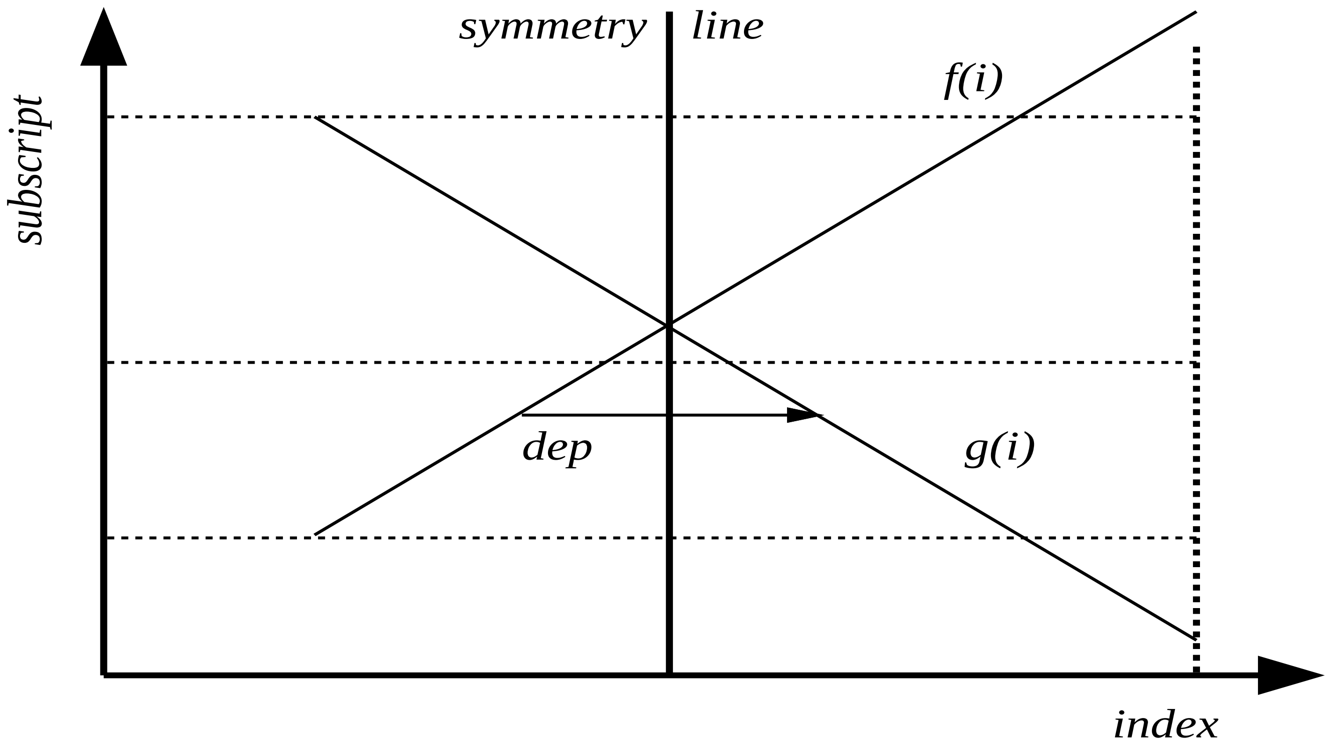
$$1 \leq N \leq N$$

Dependences!

Peel off iterations...

```
Y(1,N)=Y(1,N)+Y(N,N)
DO i = 2,N-1
   Y(i,N) = Y(1,N)+Y(N,N)
ENDDO
Y(N,N)=Y(1,N)+Y(N,N)
```

Weak Crossing SIV



$i = \frac{c_2 - c_1}{2a_1}$

Dependence only if i int. or int. +.5 and $L \leq i \leq U$

Eliminated by loop splitting

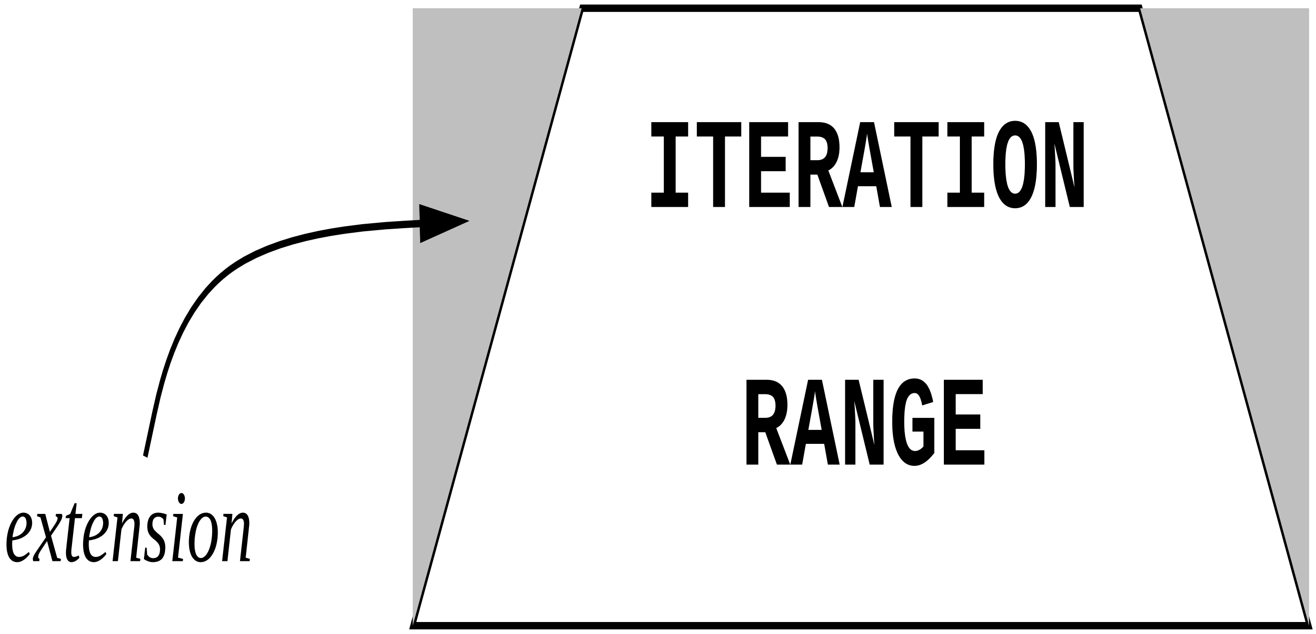
```
DO i = 1,N
   A(i) = A(N+1-i)+C
ENDDO
```

$$i = \frac{N+1-0}{2}$$

```
DO i = 1,(N+1)/2
   A(i) = A(N+1-i)+C
ENDDO
DO i = (N+1)/2+1,N
   A(i) = A(N+1-i)+C
ENDDO
```

Rectangularize the region

$\Rightarrow$ Simple conservative test



Linear Diophantine equation

$$a_1 i + a_0 = b_1 i' + b_0$$

$$a_1 x - b_1 y = b_0 - a_0$$

$$GCD(u, v) \Rightarrow \langle gcd, n_u, n_v \rangle$$

$$n_u u + n_v v = gcd(u, v)$$

GCD generates Diophantine solutions

$$\begin{aligned} GCD(a_1, -b_1) &= \langle g, n_a, n_b \rangle \\ x_k &= n_a(\frac{b_0 - a_0}{g}) + k\frac{b_1}{g} \\ y_k &= n_b(\frac{b_0 - a_0}{g}) + k\frac{a_1}{g} \end{aligned}$$

Dependences are solutions in range

Algorithm generates direction vectors

ZIV, SIV relatively straightforward

Linear SIV common case

Can we handle MIV?

Yes, but not so well...

Complex linear mappings: $Z^m \rightarrow Z$

Linear Diophantine equations

- Restricted range
- NP complete

Reduce complexity conservatively

- Search for real solutions
- Use simple analysis

$a_0 + a_1x_1 + \cdots + a_nx_n = b_0 + b_1x_1 + \cdots + b_nx_n$

Dependence only if

$gcd(a_1, \cdots, a_n, b_1, \cdots, b_n)$ divides $b_0 - a_0$

Nowhere near exact!

Banerjee Test

- Better MIV testing
- Analytic method

Delta Test

- Handles coupled subscripts efficiently
- Usually exact